Parts of a Model

- Descriptions
  - Object Description:
    - Type
    - Composition
    - Object variables – Represent intrinsic properties of the object that have fixed values
  - Process Description:
    - Reference system
    - State variables – Represent intrinsic properties with values that vary with time
    - Often useful to use graphical methods
  - Interaction Description
    - Type and agent
    - Interaction variables – Represents the interaction of some external object (called an agent) with the object being modeled
    - Often useful to use diagrams
- Formulations
  - Dynamical Laws – Mathematical equation(s) that determine(s) the time evolution of state variables
  - Interaction Laws – Mathematical equation(s) that express(es) interaction variables as functions of state variables
- Ramifications
  - Linguistic – Written and verbal communication about system and structure
  - Computational – Use of a computer program to encode system and structure

Rigid Body Rotation → Model so far...

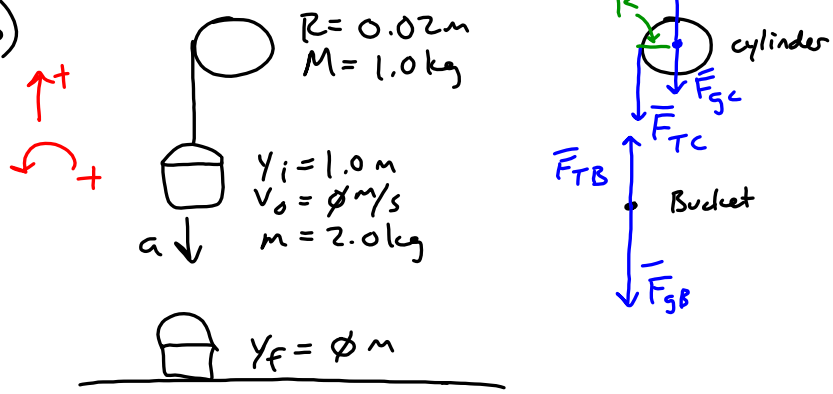
- Descriptions
  - Object Description
    - Object variables
      - Center of mass
      - Moment of inertia
      - Radius
  - Process Description
    - State variables
      - Angle
      - Angular velocity
      - Angular acceleration
      - Rotational Kinetic energy
  - Interaction Description
    - Interaction variables
      - Torque
      - Force
  - Diagrams
    - Force diagram
    - Free-body diagram
    - Energy chart (LOL diagram)

- Formulations:
  - Interaction Laws
    - $\bar{\tau} = \bar{r} \times \bar{F}$
    - $\bar{\alpha} = \frac{\sum \bar{\tau}}{I}$
    - $I = \int r^2 dm = \sum m r^2$
    - $x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$
    - $K = \frac{1}{2} I \omega^2$

- Ramifications:
  - Every object has a center of mass, but this point may not be in the geometric center of the object.
  - Moment of inertia of an object is related to the shape and orientation of the object.
  - Total kinetic energy of an object is the sum of translational kinetic energy and rotational kinetic energy.
  - The torque an object experiences is related to where and how forces are applied.

Practice - Rotational Dynamics

3)



$\Sigma \bar{F}_B = ma_B$

$F_{TB} - F_{gB} = ma_B$

$\Sigma \bar{\tau}_c = I_c \alpha_c$

$RF_{TC} = \frac{1}{2}MR^2 \alpha_c$

$F_{TB} = F_{TC}$

$\alpha_c = \boxed{\frac{2F_{TC}}{MR}}$

$-a_B = \alpha R$

$a_B = -\alpha_c R$

$= -\frac{2F_{TC}}{MR} R$

$a_B = -\frac{2F_{TC}}{M}$

$F_{TC} = \boxed{-\frac{1}{2}Ma_B}$

$-\frac{1}{2}Ma_B - ma_g = ma_B$

$-ma_g = \frac{1}{2}Ma_B + ma_B$

$-ma_g = a_B\left(\frac{1}{2}M + m\right)$

$a_B = \frac{-ma_g}{\frac{1}{2}M + m} = -7.84\,m/s^2$

$\Delta y = v_i t + \frac{1}{2}a_B t^2$

$t = \sqrt{\frac{2\Delta y}{a_B}} = 0.50\,s$