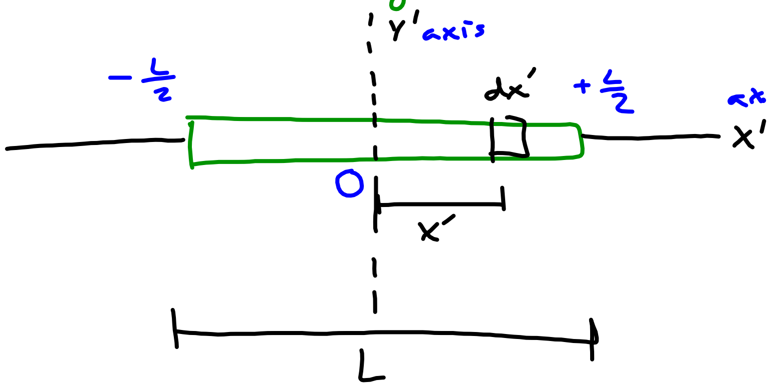
Moments of Inertia

- Rigid Object: $I = \int \rho r^2 dV$

$\rho \rightarrow$ density

- Uniform Rigid Rod



$$I = \int r^2 \, dm$$

$dm = \lambda \, dx'$

$\lambda \rightarrow \frac{\text{mass}}{\text{unit length}}$

$= \frac{M}{L} dx'$

$r = x'$

$$= \int_{-L/2}^{+L/2} (x')^2 \frac{M}{L} dx'$$

$$= \frac{M}{L} \int_{-L/2}^{+L/2} (x')^2 dx'$$

$$= \frac{M}{L} \left[\frac{x^3}{3} \right]_{-L/2}^{+L/2}$$

$$= \frac{M}{L} \left[\frac{\left(\frac{L}{2}\right)^3}{3} - \frac{\left(-\frac{L}{2}\right)^3}{3} \right]$$

$$= \frac{1}{12} M L^2$$

- Solid Cylinder

$$I = \int r^2 \, dm \qquad r = r \checkmark$$

$$dm = \rho \, dV$$

$$= \int r^2 \rho L (2\pi r) dr \qquad = \rho L (2\pi r) dr$$

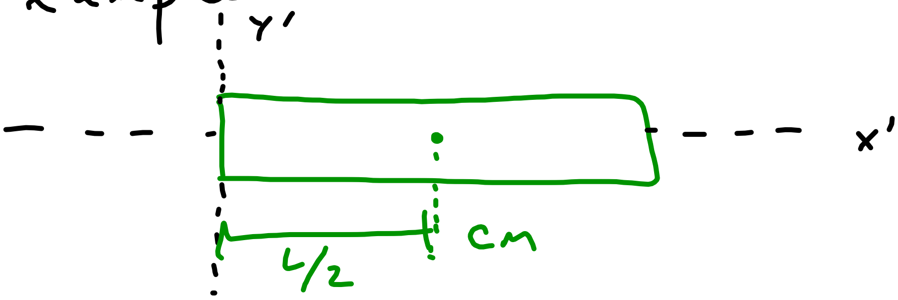
....

$$= \frac{1}{2} M R^2$$

- Parallel Axis Theorem

$$I = I_{cm} + MD^2$$

- Example:



$$I_{cm} = \frac{1}{12}ML^2$$

$$I = I_{cm} + MD^2$$

$$= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{3}ML^2$$

ROTATIONAL KINETIC ENERGY

- Equation: $K_R = \frac{1}{2} I \omega^2$

- Example: 10.10

$$K_R = \frac{1}{2} I \omega^2 \qquad I = Ma^2 + Ma^2$$
$$= 2Ma^2$$
$$= Ma^2 \omega^2$$

- Work - Rotational Kinetic Energy Theorem

$$W = \Delta K_R$$
$$W = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$