Disk of Charge

$$\vec{r} = \langle 0, 0, z \rangle$$

$$|\vec{r}| = z$$

$$\hat{r} = \langle 0, 0, 1 \rangle$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{(\Delta q)\, z}{(r^2+z^2)^{3/2}} \langle 0, 0, 1 \rangle$$

$$\Delta q = Q \frac{(\text{area of ring})}{(\text{area of disk})}$$

$$= \frac{Q\, 2\pi r\, \Delta r}{\pi R^2}$$

$$\Delta\vec{E} = \frac{1}{2 \cancel{4\pi}\varepsilon_0} \frac{\left(Q \frac{\cancel{2\pi} r\, \Delta r}{\pi R^2}\right)}{(r^2+z^2)^{3/2}} \langle 0, 0, 1 \rangle$$

$$\Delta E_z = \frac{1}{2\varepsilon_0}\left(\frac{Q}{\pi R^2}\right)\frac{z\, r\, \Delta r}{(r^2+z^2)^{3/2}}$$

$$E_z = \frac{1}{2\varepsilon_0}\left(\frac{Q}{\pi R^2}\right) z \int_0^R \frac{r\, dr}{(r^2+z^2)^{3/2}}$$

$$E_z = \frac{1}{2\varepsilon_0}\left(\frac{Q}{\pi R^2}\right)\left[1 - \frac{z}{(R^2+z^2)^{1/2}}\right]$$

Two 10-cm-diameter charged rings face each other, 20 cm apart. Both rings are charged to +20 nanocoulombs. What is the electric field strength at

(a) the midpoint between the two rings, and 0 N/C

(b) the center of the left ring? 4100 N/C