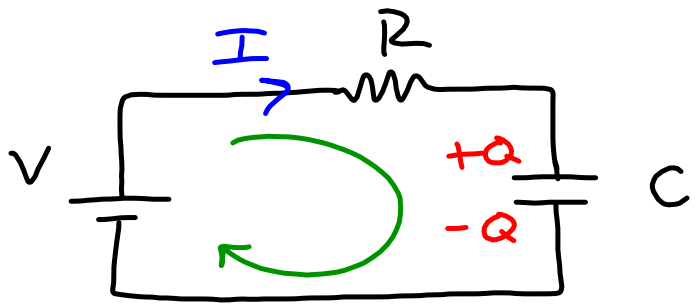
RC Circuit Derivation



(Battery) − (Resistor) − (capacitor) = ∅

$$V_B - V_R - V_c = \emptyset$$

$$+V - IR - \frac{Q}{C} = \emptyset$$

Final State of charged capacitor

$$Q = VC$$

Initial State:

$$I = \frac{1}{R}\left(V - \frac{Q}{C}\right)$$

$$\frac{dQ}{dt} = \frac{1}{R}\left(V - \frac{Q}{C}\right)$$

$$\frac{dQ}{dt} = \frac{V}{R} - \frac{1}{RC}Q$$

$$\frac{dQ}{dt} = \boxed{\frac{V}{R}} - \frac{1}{RC}Q$$

constant

$$I = I_0 e^{-at}$$

time

a = constant

I_0 → initial current

I → current at some later time

$$\frac{d}{dt}\left[I = \frac{dQ}{dt} = \frac{V}{R} - \frac{1}{RC}Q\right]$$

$\frac{V}{R}$: constant

$$\frac{dI}{dt} = -\frac{1}{RC}\frac{dQ}{dt}$$

$$\frac{dQ}{dt} = I$$

$$\frac{dI}{dt} = -\frac{1}{RC}I$$

$$\frac{dI}{dt} = -\frac{1}{RC}\left(I_0 e^{-at}\right)$$

$$\frac{d}{dt}\left(I_0 e^{-at}\right) = -\frac{1}{RC}\left(I_0 e^{-at}\right)$$

$$+a\left(I_0 e^{-at}\right) = +\frac{1}{RC}\left(I_0 e^{-at}\right)$$

$$a = \frac{1}{RC}$$

$$I = I_0 e^{-t/RC}$$