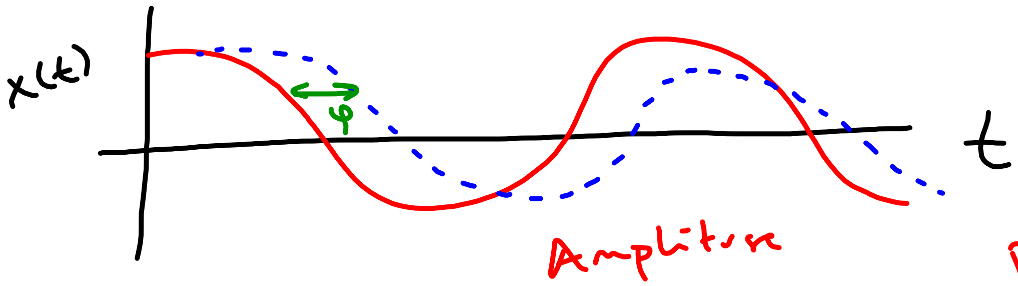
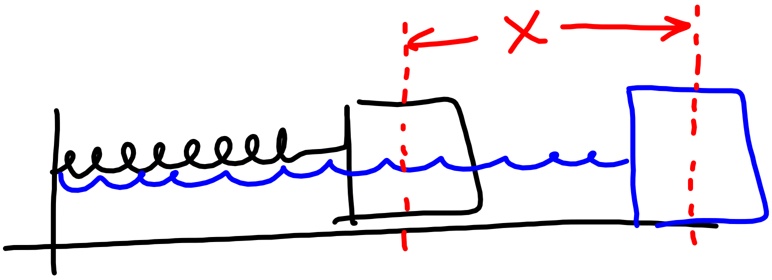
Oscillations → Particle in Simple Harmonic Motion



Amplitude phase shift

$$x(t) = A\cos(\omega t + \varphi)$$



$$F = -kx$$

$$\Sigma F = ma$$

$$F_s = ma$$

$$-kx = ma$$

$$a = -\frac{k}{m}x$$

k = spring constant

m = mass of block

a = acceleration (linear)

x = displacement from resting position

$$v = \frac{dx}{dt}$$

$$a = \frac{d^2x}{dt^2} \qquad a = -\frac{k}{m}x$$

$$a = \frac{dv}{dt}$$

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$= \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

Solutions to this equation must be repeating functions → sines or cosines (technically, $e^{i\theta}$)

$$i = \sqrt{-1}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Euler's equation

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad x(t) = A\cos(\omega t + \varphi)$$

$$\frac{dx}{dt} = A\frac{d}{dt}\left[\cos(\omega t + \varphi)\right]$$

$$= A\left[-\sin(\omega t + \varphi)\right]\omega$$

$$= -\omega A \sin(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -\omega A \frac{d}{dt}\left[\sin(\omega t + \varphi)\right]$$

$$= -\omega A\left[\cos(\omega t + \varphi)\right]\omega$$

$$= -\omega^2 A \cos(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$-\omega^2 A\cos(\omega t + \varphi) = -\omega^2 A\cos(\omega t + \varphi) \checkmark$$

$$T = \frac{2\pi}{\omega}$$ period $= \frac{2\pi}{\omega}$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

ω = angular frequency $\left[\frac{rad}{s}\right]$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$T \rightarrow$ period $\rightarrow$ time it takes to complete one cycle

$f \rightarrow$ frequency $\rightarrow$ number of cycles per second

$\left[\frac{1}{s} \equiv \text{Hertz} \equiv \text{Hz}\right]$

velocity $v = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi)$

max velocity when $|\sin(\frac{\pi}{2} \text{ or } -\frac{\pi}{2})| = 1$

$$V_{max} = \omega A = \sqrt{\frac{k}{m}}\, A$$

acceleration $a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \varphi)$

max acceleration when $|\cos(\emptyset \text{ or } \pi)| = 1$

$$a_{max} = \omega^2 A = \frac{kA}{m}$$