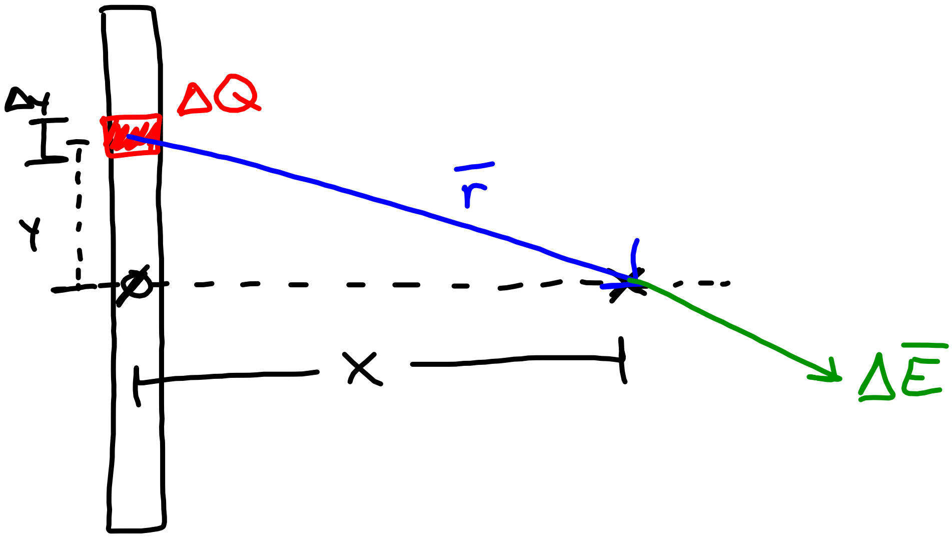
ELECTRIC FIELDS OF CHARGE DISTRIBUTIONS

Uniformly Charged Thin Rod

Process to solve:

1. Divide object into small pieces.
2. Choose an origin and axis.
3. Add the electric field contributions from each piece.
4. Check that result is physically correct.



$$\bar{r} = \langle \text{observed} \rangle - \langle \text{source} \rangle$$
$$= \langle x, \emptyset, \emptyset \rangle - \langle \emptyset, y, \emptyset \rangle$$
$$= \langle x, -y, \emptyset \rangle$$

$$|\bar{r}| = \sqrt{x^2 + (-y)^2 + (\emptyset)^2}$$
$$= \left[x^2 + (-y)^2 \right]^{1/2}$$

$$\hat{r} = \frac{\bar{r}}{|\bar{r}|} = \frac{\langle x, -y, \emptyset \rangle}{\left[(x)^2 + (-y)^2 \right]^{1/2}}$$

$$|\Delta \bar{E}| = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{|\bar{r}|^2} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{[x^2+(-y)^2]}$$

magnitude direction

$$\Delta \bar{E} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{|\bar{r}|^2}}\ \boxed{\hat{r}}$$

$$\Delta \bar{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{[x^2+(-y)^2]}\ \frac{\langle x, -y, \phi \rangle}{[x^2+(-y)^2]^{1/2}}$$

$$\Delta \bar{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{[x^2+(-y)^2]^{3/2}}\ \langle x, -y, \phi \rangle$$

$$\Delta E_x = \frac{1}{4\pi\varepsilon_0} \frac{x\,\Delta Q}{(x^2+y^2)^{3/2}}$$

$$\Delta E_y = \frac{1}{4\pi\varepsilon_0} \frac{-y\,\Delta Q}{(x^2+y^2)^{3/2}}$$

$$\Delta E_z = \emptyset$$

$$\Delta Q = \left(\frac{\Delta y}{L}\right) Q$$

small piece of charge

small piece of whole rod

total charge

$$\Delta E_x = \frac{1}{4\pi\varepsilon_0} \frac{Q}{L} \frac{x}{(x^2+y^2)^{3/2}} \Delta y$$

$$\Delta E_y = \frac{1}{4\pi\varepsilon_0} \frac{Q}{L} \frac{-y}{(x^2+y^2)^{3/2}} \Delta y$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L} x \int_{-L/2}^{+L/2} \frac{1}{(x^2+y^2)^{3/2}}\,dy$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{L} x \left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-L/2}^{+L/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{x\left(x^2+\left(\frac{L}{2}\right)^2\right)^{1/2}}\right]$$

replace x with r

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2+\left(\frac{L}{2}\right)^2}}\right]}$$

$E_y = \emptyset \rightarrow$ from integral

Uniformly Charged Thin Ring

$$\bar{r} = \langle \text{observed} \rangle - \langle \text{source} \rangle$$

$$= \langle \emptyset, \emptyset, z \rangle - \langle R\cos\theta, R\sin\theta, \emptyset \rangle$$

$$= \langle -R\cos\theta, -R\sin\theta, z \rangle$$

$\cos^2\theta + \sin^2\theta = 1$

$$|\bar{r}| = \sqrt{(-R\cos\theta)^2 + (-R\sin\theta)^2 + z^2}$$

$$= (R^2 + z^2)^{1/2}$$

$$\hat{r} = \frac{\bar{r}}{|\bar{r}|} = \frac{\langle -R\cos\theta, -R\sin\theta, z \rangle}{(R^2 + z^2)^{1/2}}$$

$$|\Delta \bar{E}| = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{|\bar{r}|^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{(R^2+z^2)}$$

$$\Delta \bar{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{(R^2+z^2)}\frac{\langle -R\cos\theta, -R\sin\theta, z\rangle}{(R^2+z^2)^{1/2}}$$

$$\Delta Q = Q\left(\frac{\Delta\theta}{2\pi}\right)$$

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\pi}\frac{\Delta\theta}{(R^2+z^2)}\frac{\langle -R\cos\theta, -R\sin\theta, z\rangle}{(R^2+z^2)^{1/2}}$$

$$\left.\begin{array}{l}\Delta E_x = \emptyset \\ \Delta E_y = \emptyset\end{array}\right]$$ from symmetry of situation

$$\Delta E_z = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\pi}\frac{z}{(R^2+z^2)^{3/2}}\Delta\theta$$

$$E_z = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\pi}\frac{z}{(R^2+z^2)^{3/2}}\int_{\emptyset}^{2\pi} d\theta$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{(R^2+z^2)^{3/2}}}$$