Rigid Body Motion Model in AP Physics C: Mechanics

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Curriculum Plan

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Literature Review

AP Physics C: Mechanics is an upper-level physics course offered to high school students, and it is the equivalent of the first-semester calculus-based physics course at many universities (College Board, 2016). Rotation is arguably the most difficult aspect of AP Physics C: Mechanics because students have many interesting preconceptions about rotation but lack models to successfully analyze situations. Rotation has two parts—point particles and rigid bodies—each requiring a model. Previously in the course, students have studied the rotation of a point particle, learning many useful concepts and creating a complete model. This unit extends rotation from a point particle to rigid bodies, and this shift enables students to learn concepts and applications unique to rigid body rotation.

The problem for this curriculum plan is to help each student create a rigid body rotation mental model that matches the accepted scientific rigid body rotation conceptual model and is consistent with rigid body rotation in the physical world. Students have generated models for other content in AP Physics C: Mechanics, but this problem focuses on the rigid body motion model. Rigid body rotation is approximately 15% of the AP Physics C: Mechanics course (College Board, 2014) and is consistently included in one of the three free-response questions on the AP Physics C: Mechanics exam. Rigid body rotation questions are approximately 8 out of the 35 total questions in the multiple-choice section, and students often have the lowest percent of correct answers on these questions. In addition, the author taught AP Physics C: Mechanics in the 2015-2016 school year, and rigid body rotation was the most difficult unit for students to comprehend. One reason for these difficulties is a lack of familiarity with the underlying concepts of rigid body rotation, leading students to possess many ideas that are inconsistent with the accepted scientific conceptual model and physical world. Although rotating tires and fan blades are a typical sight for many students, lower-level physics courses do not discuss the mathematical and physical ideas about rigid body rotation. Students learn new mathematics and apply prior mathematical knowledge in new ways, complicating students' understanding about the physics of rigid body rotation. Therefore, students need many opportunities to develop and refine their mental model of rigid body rotation, and this curriculum plan uses 21 class periods—97 minutes per class period—to cycle through many aspects of rigid body rotation.

Physics Education Research has identified several pedagogies as best-practices for physics education, including Modeling Instruction (Jackson, Dukerich, & Hestenes, 2008). Modeling Instruction is a constructivist, student-centered version of model-based inquiry whereby students organize content into conceptual mental models and work to align their conceptual mental model with the accepted scientific conceptual model and physical world. The work to change students' mental models utilizes the eight science and engineering practices (SEPs) established by the National Research Council in *A Framework for K-12 Science Education: Practices, Crosscutting Concepts, and Core Ideas* (2012), which are

1. Asking questions (for science) and defining problems (for engineering)
2. Developing and using models
3. Planning and carrying out investigations
4. Analyzing and interpreting data
5. Using mathematics and computational thinking
6. Constructing explanations (for science) and designing solutions (for engineering)
7. Engaging in argument from evidence
8. Obtaining, evaluating, and communicating information. (p. 42)

Modeling Instruction places the most emphasis on the second SEP, "developing and using models," and students use the other seven SEPs to create, refine, and determine the limits of their mental model.

Constructivism is a theory of learning that provides underlying ideas for Modeling Instruction. "Constructivism's central idea is that human knowledge is *constructed*, that learners build new knowledge upon the foundation of previous learning" (Kanselaar, 2002, p. 1). This theory is not a single idea; rather it encompasses beliefs about the nature of reality, learning and cognition, and pedagogy (Kanselaar, 2002). Two major historical strands were developed in the 20th century—cognitive constructivism by Jean Piaget and social-cultural constructivism by Lev Vygotsky—and these have significant differences when discussing the stages of learning. Modeling Instruction is unconcerned with the differences, and focuses more attention on psychological ideas about learning and cognition and educational ideas about pedagogy.

The theoretical base of Modeling Instruction is grounded in constructivism, but focuses on the idea of a model as its thematic core. "A **model** is a **representation of structure** in a given **system** [emphasis in original]. A system is a set of related objects, which may be real or imaginary, physical or mental, simple or composite. The structure of a system is a set of relations among its objects" (Hestenes, 2016, p. 16). A connection between the physical, mental, and conceptual worlds is encapsulated in the modeling theory of cognition, and the theory rests on a distinction between mental models and conceptual models. Mental models are unique to an individual, and they represent the way the individual interacts with the physical world and conceptual models for a given topic. Conceptual models are the accepted scientific model for a given topic because the model explains and predicts the way the physical world behaves. Conceptual models are subject to revision given new information about the way the world works, and the process of revising conceptual models is foundational to science. Links between the three worlds highlight how they interact (see Figure 1), providing a theoretical basis that helps students align their rigid body rotation mental model with information from the physical world and accepted scientific rigid body rotation conceptual model.

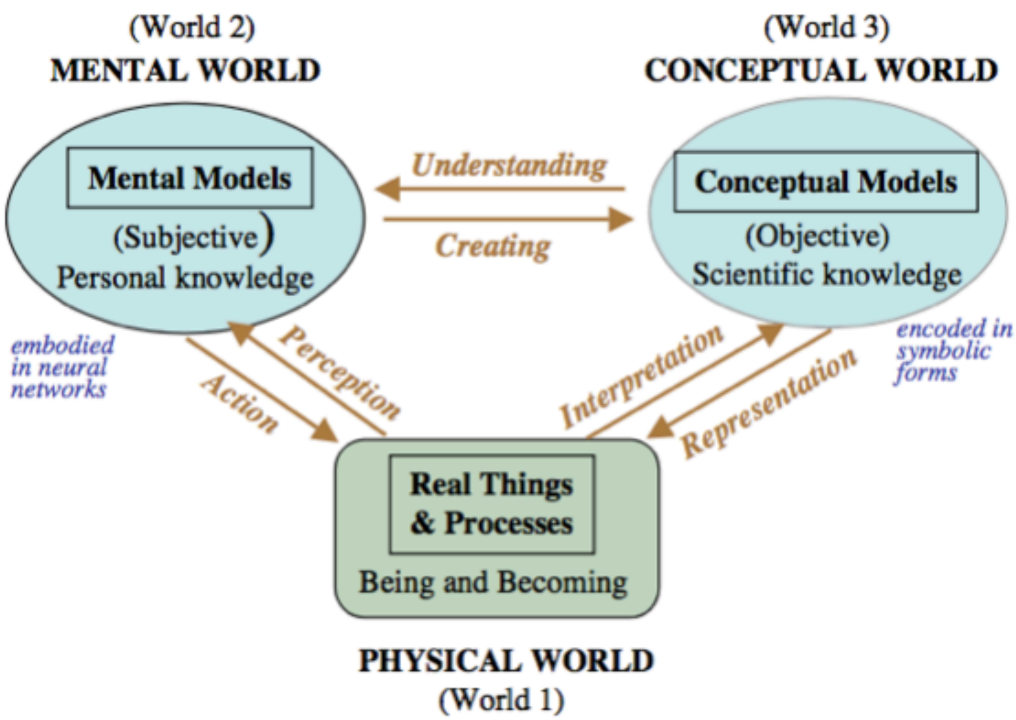

Figure 1. Relationships between the physical, mental, and conceptual worlds (Hestenes, 2006, p. 10). From "Notes for a Modeling Theory of Science, Cognition and Instruction" by D. Hestenes, 2006, *Proceedings of the 2006 GIREP Conference*. Copyright 2006 by David Hestenes. Reprinted with permission.

The pedagogical framework for Modeling Instruction is the modeling cycle, patterned after the learning cycle developed by Robert Karplus and others (Lawson, Abraham, & Renner, 1989; Karplus, 1969). The modeling cycle has two distinct parts: Model development, in which students perform a paradigm laboratory and engage in discussions to create an initial mental model related to the physical world; and model deployment, during which students manipulate and test the model to ensure alignment with the physical world and accepted scientific conceptual model. An initial model arises from students asking questions of their world, planning and carrying out investigations, and analyzing and interpreting data from the investigations. During analysis and conclusions about the initial model, students use mathematical and computation thinking, construct explanations, engage in arguments from the data, and communicate information about the model. After completing an initial model, students deploy the initial model in physical and conceptual investigations to determine the limits of and refine the model, creating a complete model. Students practice applying the complete model during this process and synthesize relevant scientific content within the complete model. When the complete model is shown to be incapable of accurately explaining the results of an investigation, students develop a new initial model and follow the process to create another complete model. Assessments in the form of mathematical or computational problems, quizzes, and additional laboratories are used formatively, and the modeling cycle is completed with a laboratory practicum and summative unit assessment. One major aspect that separates Modeling Instruction from other instructional varieties is whiteboarding. The whiteboards are 24" x 32" erasable pieces that students use during all parts of the modeling cycle, providing opportunities to make thinking visible around scientific content and processes. When performing laboratories, students record, graph, and analyze data on their whiteboard for presentation during the post-lab discussion. Having visible information from all groups allows students to compare, contrast, and question data and analysis easily, creating a robust discussion about the results.

Modeling Instruction has been implemented most frequently in high school physics courses, with over 3,000 teachers participating in summer workshops from 1995 to the present. The Force Concept Inventory (FCI) "has become the most widely used and influential instrument for assessing the effectiveness of introductory physics instruction" (Jackson, Dukerich, & Hestenes, 2008, p. 15), and the aggregate of these scores shows a large effect of Modeling Instruction on the achievement of students in physics courses versus a much smaller effect from traditional instruction. Figure 2 shows the difference in student achievement on the FCI for instructional type, and "Expert Modelers" obtain the highest gains on the FCI.

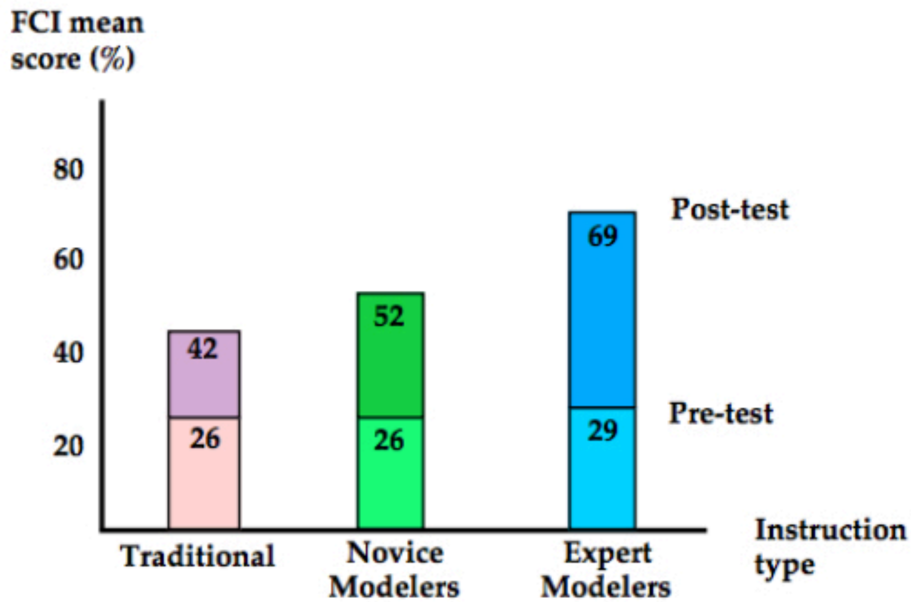

Figure 2. FCI mean pretest and posttest scores under different instruction types. From "Notes for a Modeling Theory of Science, Cognition and Instruction" by D. Hestenes, 2006, *Proceedings of the 2006 GIREP Conference*. Copyright 2006 by David Hestenes. Reprinted with permission.

The seminal study for Modeling Instruction was performed by Malcolm Wells with high school physics students in Arizona, and he found a modeling course had a 34% increase between the pretest and posttest versus a 13% increase from a traditional course (Wells, Hestenes, & Swackhamer, 1995). A recent study on the effect of Modeling Instruction in a Louisiana high school physics classroom was conducted by Mark Arseneault (2014), and he found the courses with Modeling Instruction had a 17% increase between the pretest and posttest versus a 10% increase of the traditional courses. Data from other studies on Modeling

Instruction have consistently shown a higher increase in student performance on the FCI and other assessments than other instructional methods (Brewe, 2002; Brewe, 2008; Hake, 1998; Melendez & Wirth, 2001; O'Brien & Thompson, 2009). In 2001, "a U.S. Department of Education Expert Panel in Science recognized the Modeling Instruction Program as one of only two exemplary K-12 science programs out of 27 programs evaluated" (Jackson, Dukerich, & Hestenes, 2008, p. 17). Although research on Modeling Instruction has been performed for over 20 years, there are places for further research. Most of the published articles in physics have been at the high school level, with several studies performed at the collegiate level. However, no published studies exist for either AP Physics 1 and 2 or AP Physics C: Mechanics and Electricity and Magnetism, so this curriculum plan is a novel document within the research base. In addition, few published studies exist specifically concerning rigid body rotation, which makes this curriculum plan valuable for other AP Physics C: Mechanics teachers.

The curriculum plan for rigid body rotation includes the theoretical and practical aspects of constructivism, modeling theory, and Modeling Instruction, helping students generate a rigid body rotation mental model that is consistent with the rigid body rotation conceptual model and rigid body rotation in the physical world. Students perform paradigm laboratory work to create an initial model, then refine and determine the limits of the model through further laboratory investigations. These investigations require students to produce their own procedures and determine variables to change and keep constant, while emphasizing the relationship with the rigid body model. Students apply mathematical thinking through practice problems, discussing the setup, procedures, and solutions to the problems. Students create a model summary at several points throughout the series of lessons to solidify their understanding of the rigid body rotation model and determine the alignment of their mental model with the accepted conceptual model and physical world. Formative assessments provide feedback to students about their alignment of the models, and a summative assessment supplies information about the overall understanding of rigid body rotation. With the alignment of this curriculum plan to the theory and pedagogy of Modeling Instruction, students should perform well on the AP Physics C: Mechanics exam and have a substantial mental model for future studies of rigid body rotation.

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Curriculum Plan

Part I: Instructional Goals

Instructional goals serve as a link between curriculum, instruction, and assessment, providing information about the expectations in the unit. For this curriculum plan, the instructional goals are:

1. Students will develop a coherent model for rigid body rotation.
2. Students will perform calculations in rigid body rotation.
3. Students will utilize laboratory skills to collect data, interpret data, and create conclusions.

Goal 1 was selected to ensure alignment with Modeling Instruction theory, which is to use inquiry experiences to create a mental model that is close to the accepted scientific conceptual model. It also serves as the driving force behind the activities and practice because all activities and practice should be helping students create, refine, or define the limits of the rigid body motion model. Goals 2 and 3 were selected to help students develop skills for this course and future science courses. Goal 2 focuses on calculations within the rigid body motion model, and students need familiarity with a wide range of problems to understand their connection to the model and themes within each problem. This will help students perform well on the AP Physics C: Mechanics exam and provide techniques for future science courses. Goal 3 focuses on laboratory work, which is critical when developing a model; the model must make accurate predictions of real-world application, and laboratory work is how students test the model to determine the accuracy of the predictions. In addition, students may take future courses or work in careers where laboratory work is expected, and this goal helps give students a base for performing experiments.

Part II: Sequence of Lessons

1. Center of mass, moment of inertia, rotational energy
    a. Lab – Center of mass (10/21)
    b. Discussion – Center of mass, moment of inertia, rotational energy (10/21)
    c. Lab – Rotational energy (10/24)
    d. Practice – Center of mass, moment of inertia, rotational energy (10/25)
    e. Practice – Center of mass, moment of inertia, rotational energy (10/26)
    f. Practice – Center of mass, moment of inertia, rotational energy (10/27)
    g. Quiz – Center of mass, moment of inertia, rotational energy (10/27)
2. Rotational statics and dynamics
    a. Lab – Rotational statics (10/28)
    b. Discussion – Rotational statics (10/31)
    c. Practice – Rotational statics (10/31)
    d. Lab – Rotational dynamics (11/1)
    e. Discussion – Rotational dynamics (11/2)
    f. Practice – Rotational dynamics (11/2)
    g. [Practice – AP Problems (11/3 and 11/4; I was attending the South Carolina Science Council]
    h. Review – Model creation (11/7)
    i. Practice – Rotational dynamics (11/7)
    j. Practice – Rotational statics and dynamics (11/9)
    k. Practice – Rotational statics and dynamics (11/10)
    l. Quiz – Rotational statics and dynamics (11/10)
3. Angular Momentum
    a. Lab – Angular momentum (11/11)
    b. Discussion – Angular momentum (11/14)
    c. Practice – Angular momentum (11/14)
    d. Review – Model Update (11/15)
    e. Practice – Angular momentum (11/15)
    f. Practice – Angular momentum (11/16)
    g. Quiz – Angular momentum (11/16)
4. Summative Assessment
    a. Review – Rigid Body Rotation (11/17)
    b. Test – Rigid Body Rotation (11/18)

All standards are from the College Board: https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-physics-c-course-description.pdf

Essential Question: What are the implications of rotating rigid bodies?

Lesson Plan for 2016-10-21

Class	AP Physics C: Mechanics
Title/Topic	Center of Mass, Moment of Inertia, Rotational Energy

Standards

- D.1.a.1: Identify by inspection the center of mass of a symmetrical object.
- D.1.a.2: Locate the center of mass of a system consisting of two such objects.
- E.3.a: Students should understand the analogy between translational and rotational kinematics so they can write and apply relations among the angular acceleration, angular velocity, and angular displacement of an object that rotates about a fixed axis with constant angular acceleration.

Objectives

Students should be able to:

- Determine the center of mass by calculation and experimentation for linear configurations.
- Determine the center of mass by calculation and experimentation for non-linear configurations.
- Calculate velocity of objects as they rotate about the center of mass.
- Derive an equation for the rotational kinetic energy of a non-uniform object.

Context

1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because ideas about center of mass and moment of inertia are foundational to the model for rigid body rotation.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations for the center of mass in linear and non-linear configurations and compare them to experimental results. Students also calculate the velocity of objects as they rotate about the center of mass.

3. *Students will utilize laboratory skills to collect data, interpret data, and create conclusions*. The lesson meets this goal because students are collecting data, interpreting data, and creating conclusions about the placement of the masses.

Materials

- Lab – Center of Mass
- Masses of varying sizes – 5 per group
- Rulers to align the masses in linear configurations – 1 per group
- Lids of Tupperware boxes to align the masses in non-linear configurations – 1 per group
- Whiteboards – 1 or 2 per group
- Markers and towels – class set

Safety	Handle masses appropriately

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (55 minutes) Lab – Center of Mass
	    - How does the placement of the masses change the center of mass?
	    - How does the relative mass at each placement change the center of mass?
	- (15 minutes) Group discussion of results of Lab – Center of Mass
	    - How close were your calculations and experimental determinations?
	    - What could have gone wrong in this lab?
	- (10 minutes) Discussion and Practice – Center of Mass
	    - How do we calculate the center of mass?
	    - How do we calculate the speed of each ball as it is rotating?
	- (10 minutes) Discussion – Rotational Energy and Moment of Inertia
	    - How is the total kinetic energy of an object determined?
	    - How is the moment of inertia defined?

Differentiation	- Help students who are struggling in the lab - Challenge students who complete initial configurations to create more difficult configurations - Help students who are struggling with the calculations
Assessment	- Comparison of calculations and experimentally determined location - Individual students in the whole-group discussion - Calculations on problems
References	Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-10-24

Class	AP Physics C: Mechanics
Title/Topic	Rotational Kinetic Energy
Standards	- E.3.a: Students should understand the analogy between translational and rotational kinematics so they can write and apply relations among the angular acceleration, angular velocity, and angular displacement of an object that rotates about a fixed axis with constant angular acceleration. - E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.
Objectives	Students should be able to: - Determine the angular velocity of a ball at the end of a ramp. - Determine the rotational kinetic energy of a ball at the end of a ramp. - Develop a quantitative LOL diagram for the ball rolling down the ramp.
Context	1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because rotational kinetic energy is an overarching part of the rigid body rotation model. The model utilizes multiple representations for motion and energy, and students use and LOL diagram to discuss the changes in energy as the ball rolls down the ramp. 2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations for angular velocity and rotational kinetic energy as they analyze data. 3. *Students will utilize laboratory skills to collect data, interpret data, and create conclusions*. The lesson meets this goal because students are collecting data, interpreting data, and creating conclusions about the transfer of energy as the ball moves down the ramp.
Materials	- Lab – Rotational Kinetic Energy - Ramp – 1 per group - Athletic balls (tennis, golf, lacrosse, soft golf, bouncy) – enough for the class to share - LabQuest 2 – 1 per group - Motion detector – 1 per group - Whiteboards – 1 or 2 per group - Markers and towels – class set
Safety	Handle equipment appropriately

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (80 minutes) Lab – Rotational Kinetic Energy
	    - How does the construction of the ball impact the rotational kinetic energy?
	    - Which ball has the most or least rotational kinetic energy at the bottom of the ramp?
	- (10 minutes) Group discussion of results of Lab – Rotational Kinetic Energy
	    - How do your calculations compare with the calculations of other?
	    - Which ball has the most or least rotational kinetic energy at the bottom of the ramp?
	    - What could have gone wrong in this lab?

Differentiation	- Help students who are struggling in the lab with either data collection or calculations. - Challenge students who complete lab quickly to think more deeply about rotational kinetic energy.
Assessment	- Data collection and calculations - Individual students in the whole-group discussion
References	Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-10-25

Class	AP Physics C: Mechanics
Title/Topic	Center of Mass, Moment of Inertia, Rotational Energy

Standards

- E.2.c.1: Determine by inspection which of a set of symmetrical objects of equal mass has the greatest rotational inertia.
- E.2.d.1: A collection of point masses lying in a plane about an axis perpendicular to the plane.
- E.2.d.2: A thin rod of uniform density, about an arbitrary axis perpendicular to the rod.
- E.2.d.3: A thin cylindrical shell about its axis, or an object that may be viewed as being made up of coaxial shells.
- E.2.e: Students should be able to state and apply the parallel-axis theorem.
- E.3.a: Students should understand the analogy between translational and rotational kinematics so they can write and apply relations among the angular acceleration, angular velocity, and angular displacement of an object that rotates about a fixed axis with constant angular acceleration.
- E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation.
- E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.

Objectives

Students should be able to:

- Order a set of objects from greatest to least moment of inertia.
- Calculate moments of inertia using the parallel axis theorem.
- Explain the difference in velocity between the top and bottom points of a rolling object.
- Perform rigid body rotation calculations correctly.

Context

1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are using what they have learned about center of mass, moment of inertia, and rotational energy in the last two lessons to perform correct calculations on practice problems and TIPERs.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations for the center of mass, moment of inertia, and rotational energy.

Materials

- TIPERs – B6-RT03, B6-RT06, B6-RT04
- Practice – Center of Mass, Moments of Inertia, and Rotational Energy

Safety N/A

Instructional Sequence
- (7 minutes) Greeting and general discussion
- (15 minutes) Practice - TIPERs
  - How does the configuration of the object affect the moment of inertia?
  - How does the shape of the solid object affect the moment of inertia?
- (30 minutes) Discussion – Parallel Axis Theorem and Rolling Motion
  - How does the parallel axis theorem help when calculating moments of inertia?
  - What are the velocities of the top and bottom point on an object that is rolling? Why do these points have that velocity?
- (45 minutes) Practice – Center of Mass, Moments of Inertia, and Rotational Energy
  - How do we perform the calculations correctly?
  - Is there another way to solve this problem?

Differentiation
- Help students who are struggling with the calculations
- Challenge students who are finished early to perform calculations with another method

Assessment
- TIPERs worksheet
- Calculations on problems

References
Hieggelke, C. J., Kanim, S., Maloney, D. P., & O'Kuma, T. L. (2015). *TIPERs: Sensemaking tasks for introductory physics*. Boston, MA: Pearson.

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-10-26

Class	AP Physics C: Mechanics
Title/Topic	Center of Mass, Moment of Inertia, Rotational Energy
Standards	- E.2.c.1: Determine by inspection which of a set of symmetrical objects of equal mass has the greatest rotational inertia. - E.2.d.1: A collection of point masses lying in a plane about an axis perpendicular to the plane. - E.2.d.2: A thin rod of uniform density, about an arbitrary axis perpendicular to the rod. - E.2.d.3: A thin cylindrical shell about its axis, or an object that may be viewed as being made up of coaxial shells. - E.2.e: Students should be able to state and apply the parallel-axis theorem. - E.3.a: Students should understand the analogy between translational and rotational kinematics so they can write and apply relations among the angular acceleration, angular velocity, and angular displacement of an object that rotates about a fixed axis with constant angular acceleration. - E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation. - E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.
Objectives	Students should be able to: - Perform rigid body rotation calculations correctly.
Context	1. *Students will develop a coherent model for rigid body rotation.* The lesson meets this goal because students are using what they have learned about center of mass, moment of inertia, and rotational energy to perform correct calculations on practice problems. 2. *Students will perform calculations in rigid body rotation.* The lesson meets this goal because students perform calculations for the center of mass, moment of inertia, and rotational energy.
Materials	- Practice – Center of Mass, Moments of Inertia, and Rotational Energy
Safety	N/A
Instructional Sequence	- (7 minutes) Greeting and general discussion - (90 minutes) Practice – Center of Mass, Moments of Inertia, and Rotational Energy - How do we perform the calculations correctly?

- Is there another way to solve this problem?

Differentiation
- Help students who are struggling with the calculations
- Challenge students who are finished early to perform calculations with another method

Assessment
- Calculations on problems

References
Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-10-27

Class	AP Physics C: Mechanics
Title/Topic	Center of Mass, Moment of Inertia, Rotational Energy

Standards

- E.2.c.1: Determine by inspection which of a set of symmetrical objects of equal mass has the greatest rotational inertia.
- E.2.d.1: A collection of point masses lying in a plane about an axis perpendicular to the plane.
- E.2.d.2: A thin rod of uniform density, about an arbitrary axis perpendicular to the rod.
- E.2.d.3: A thin cylindrical shell about its axis, or an object that may be viewed as being made up of coaxial shells.
- E.2.e: Students should be able to state and apply the parallel-axis theorem.
- E.3.a: Students should understand the analogy between translational and rotational kinematics so they can write and apply relations among the angular acceleration, angular velocity, and angular displacement of an object that rotates about a fixed axis with constant angular acceleration.
- E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation.
- E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.

Objectives

Students should be able to:

- Order a set of objects from greatest to least rotational kinetic energy.
- Perform rigid body rotation calculations correctly.

Context

1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are using what they have learned about center of mass, moment of inertia, and rotational energy to perform correct calculations on practice problems and TIPERs.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations for the center of mass, moment of inertia, and rotational energy.

Materials

- TIPERs – B6-RT08, B6-QRT29
- Practice – Problem on Presentation
- Quiz – Center of Mass, Moments of Inertia, Rotational Energy

Safety N/A

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (15 minutes) Practice - TIPERs
	    - How does the configuration of the object affect the moment of inertia?
	    - How does the shape of the solid object affect the moment of inertia?
	- (30 minutes) Practice – Problem on Presentation
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?
	- (45 minutes) Quiz – Center of Mass, Moments of Inertia, and Rotational Energy

Differentiation	- Help students who are struggling with the TIPERs or calculation - Challenge students who are finished early to perform calculations with another method - Accommodations on quiz as determined by 504/IEP
Assessment	- TIPERs worksheet - Calculations on problem - Quiz
References	Hieggelke, C. J., Kanim, S., Maloney, D. P., & O'Kuma, T. L. (2015). *TIPERs: Sensemaking tasks for introductory physics*. Boston, MA: Pearson. Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-10-28

Class	AP Physics C: Mechanics
Title/Topic	Rotational Statics
Standards	- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force. - E.2.a.2: Calculate the torque on a rigid object due to gravity. - E.2.b.1: State the conditions for translational and rotational equilibrium of a rigid object. - E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations.
Objectives	Students should be able to: - Determine the relationship between the force to hold a hanging mass and position away from the rotation point. - Create and interpret a graph of the data. - Perform linearization to data for easier interpretation.
Context	1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because ideas about rotational statics are foundational to rigid body rotation. 2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to determine the torque of the masses on the rotation point. 3. *Students will utilize laboratory skills to collect data, interpret data, and create conclusions*. The lesson meets this goal because students are collecting data, interpreting data, and creating conclusions about the relationship between placement position and force.
Materials	- Lab – Rotational Statics - Masses of varying sizes – 1 set per group - Ring stands – 1 per group - Rotational static apparatus – 1 per group - Spring scale – 1 per group - Demonstration simple truss – 1 per group (Frey Scientific: https://store.schoolspecialty.com/OA_HTML/ibeCCtpItmDspRte.jsp?minisite=10029&item=3006130 - Whiteboards – 1 or 2 per group - Markers and towels – class set
Safety	Handle equipment appropriately

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (65 minutes) Lab – Rotational Statics
	    - How does the placement of the masses change the force on the rotation point?
	    - How does linearizing the data help with the interpretation?
	- (15 minutes) Group discussion of results of Lab – Rotational Statics
	    - What is your interpretation of the data and analysis?
	    - What could have gone wrong in this lab?
	- (10 minutes) Discussion – Rotational Statics
	    - How does a cross product affect the way the vectors are “multiplied”?
	    - What are the conditions for rotational statics?

Differentiation	- Help students who are struggling in the lab - Challenge students who complete lab quickly to determine relationship through linearization
Assessment	- Collection of data, graphing, linearization, and interpretations - Individual students in the whole-group discussion
References	Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-10-31

Class	AP Physics C: Mechanics
Title/Topic	Rotational Statics
Standards	- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force. - E.2.a.2: Calculate the torque on a rigid object due to gravity. - E.2.b.1: State the conditions for translational and rotational equilibrium of a rigid object. - E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations. - E.4.a.1: Calculate the torque of a specified force about an arbitrary origin.
Objectives	Students should be able to: - Calculate the cross (vector) product of two vectors. - Calculate torque or associated variables for a given situation.
Context	1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with information on rotational statics that has been incorporated into the model. 2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to determine torque and other variables associated with rotational statics.
Materials	- Practice – Cross Products and Rotational Statics - TIPERs – B6-QRT07
Safety	N/A

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (80 minutes) Practice – Cross Products and Rotational Statics
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?
	- (10 minutes) TIPERs – B6-QRT07
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?

Differentiation	- Help students who are struggling with calculations - Challenge students who have finished to help their peers
Assessment	- TIPERs worksheet

- Calculations on problems

References

Hieggelke, C. J., Kanim, S., Maloney, D. P., & O'Kuma, T. L. (2015). *TIPERs: Sensemaking tasks for introductory physics*. Boston, MA: Pearson.

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-01

Class	AP Physics C: Mechanics
Title/Topic	Rotational Dynamics
Standards	- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force. - E.3.c.2: Determine the angular acceleration with which a rigid object is accelerated about a fixed axis when subjected to a specified external torque or force.
Objectives	Students should be able to: - Determine the relationship between torque and angular acceleration. - Create and interpret a graph of the data. - Compare a value from the graph to the moment of inertia of the object.
Context	1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because ideas about rotational dynamics are foundational to rigid body rotation. 2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to determine the relationship between torque and angular acceleration and moment of inertia of an object. 3. *Students will utilize laboratory skills to collect data, interpret data, and create conclusions*. The lesson meets this goal because students are collecting data, interpreting data, and creating conclusions about the relationship between torque and angular acceleration.
Materials	- Rotary motion sensor – 1 set per group - Ring stands – 1 per group - LabQuest 2 (from Vernier) – 1 per group - Rotational motion accessory kit – 1 per group - String – 1 per group - Hanging mass set – 1 per group - Whiteboards – 1 or 2 per group - Markers and towels – class set
Safety	Handle equipment appropriately

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (75 minutes) Lab – Rotational Dynamics
	    - How does the torque affect the angular acceleration?
	    - How does the slope compare to the moment of inertia of the rotating object?

- (15 minutes) Group discussion of results of Lab – Rotational Dynamics
  - What is your interpretation of the data and analysis?
  - What could have gone wrong in this lab?

Differentiation
- Help students who are struggling in the lab
- Challenge students who complete lab quickly to provide interpretation of other situations

Assessment
- Collection of data, graphing, and interpretations
- Individual students in the whole-group discussion

References

Dukerich, L. (2011). *Advanced physics with Vernier: Mechanics*. Beaverton, OR: Vernier Software and Technology. Retrieved from https://www.vernier.com/experiments/phys-am/13/rotational_dynamics/

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-02

Class	AP Physics C: Mechanics
Title/Topic	Rotational Statics
Standards	- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force. - E.2.a.2: Calculate the torque on a rigid object due to gravity. - E.2.b.1: State the conditions for translational and rotational equilibrium of a rigid object. - E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations. - E.4.a.1: Calculate the torque of a specified force about an arbitrary origin.
Objectives	Students should be able to: - Calculate torque or associated variables for a given situation.
Context	1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with information on rotational statics that has been incorporated into the model. 2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to determine torque and other variables associated with rotational statics.
Materials	- Practice – Cross Products and Rotational Statics (given to students on 2016-10-31)
Safety	N/A

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (90 minutes) Practice – Cross Products and Rotational Statics
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?

Differentiation	- Help students who are struggling with calculations - Challenge students who have finished to help their peers
Assessment	- Calculations on problems
References	Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-03

Class	AP Physics C: Mechanics
Title/Topic	Rotational Statics and Dynamics (Note: The author was absent for this lesson to attend the 2016 South Carolina Science Council annual convention.)

Standards

- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force.
- E.2.a.2: Calculate the torque on a rigid object due to gravity.
- E.2.b.1: State the conditions for translational and rotational equilibrium of a rigid object.
- E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations.
- E.2.e: Students should be able to state and apply the parallel-axis theorem.
- E.3.c.2: Determine the angular acceleration with which a rigid object is accelerated about a fixed axis when subjected to a specified external torque or force.
- E.3.c.3: Determine the radial and tangential acceleration of a point on a rigid object.
- E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation.
- E.3.c.5: Analyze problems involving strings and massive pulleys.
- E.3.d.1: Write down, justify and apply the relation between linear and angular velocity, or between linear and angular acceleration, for an object of circular cross-section that rolls without slipping along a fixed plane, and determine the velocity and acceleration of an arbitrary point on such an object.
- E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.
- E.4.a.1: Calculate the torque of a specified force about an arbitrary origin.

Objectives

Students should be able to:

- Determine the effect of multiple torques on the rotation of an object.
- Perform calculations on rotational static problems.
- Perform calculations on rotational dynamics problems.

Context

1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with information on rotational statics and dynamics that have been incorporated into the model.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations on rotational statics and dynamics problems.

Materials

- TIPERs – B6-CT10, B6-RT11, B6-RT12, B6-QRT13, B6-RT16, B6-CT17, B6-CT18, B6-RT19, B6-RT21, B6-RT22, B6-RT23, B6-RT24, B6-RT25, B6-RT26
- AP Physics C: Mechanics Free-Response Problems

Safety

N/A

Instructional Sequence

- (7 minutes) Greeting and general discussion
- (45 minutes) TIPERs – B6-CT10, B6-RT11, B6-RT12, B6-QRT13, B6-RT16, B6-CT17, B6-CT18, B6-RT19, B6-RT21, B6-RT22, B6-RT23, B6-RT24, B6-RT25, B6-RT26
    - How do we perform the calculations correctly?
    - Is there another way to solve this problem?
- (45 minutes) AP Physics C: Mechanics Free-Response Problems
    - How do we perform the calculations correctly?
    - Is there another way to solve this problem?

Differentiation

- Help students who are struggling with calculations
- Challenge students who have finished to help their peers

Assessment

- Calculations on problems

References

Hieggelke, C. J., Kanim, S., Maloney, D. P., & O'Kuma, T. L. (2015). *TIPERs: Sensemaking tasks for introductory physics*. Boston, MA: Pearson.

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-04

Class	AP Physics C: Mechanics
Title/Topic	Rotational Statics and Dynamics (Note: The author was absent for this lesson to attend the 2016 South Carolina Science Council annual convention.)

Standards

- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force.
- E.2.a.2: Calculate the torque on a rigid object due to gravity.
- E.2.b.1: State the conditions for translational and rotational equilibrium of a rigid object.
- E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations.
- E.2.e: Students should be able to state and apply the parallel-axis theorem.
- E.3.c.2: Determine the angular acceleration with which a rigid object is accelerated about a fixed axis when subjected to a specified external torque or force.
- E.3.c.3: Determine the radial and tangential acceleration of a point on a rigid object.
- E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation.
- E.3.c.5: Analyze problems involving strings and massive pulleys.
- E.3.d.1: Write down, justify and apply the relation between linear and angular velocity, or between linear and angular acceleration, for an object of circular cross-section that rolls without slipping along a fixed plane, and determine the velocity and acceleration of an arbitrary point on such an object.
- E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.
- E.4.a.1: Calculate the torque of a specified force about an arbitrary origin.

Objectives

Students should be able to:
- Complete the levers game with knowledge of rotational statics and dynamics.
- Perform calculations on rotational static problems.
- Perform calculations on rotational dynamics problems.

Context

1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with information on rotational statics and dynamics that have been incorporated into the model.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations on rotational statics and dynamics problems.

Materials	- AP Physics C: Mechanics Free-Response Problems - Computers – 1 per student - Vectorpark – Levers (http://www.vectorpark.com/levers/)
Safety	Handle the computers properly

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (45 minutes) AP Physics C: Mechanics Free-Response Problems
	  - How do we perform the calculations correctly?
	  - Is there another way to solve this problem?
	- (45 minutes) Vectorpark – Levers
	  - How does each part affect the balance of the whole?
	  - What are the relative masses for each part?

Differentiation	- Help students who are struggling with calculations - Challenge students who have finished to help their peers
Assessment	- Calculations on problems - Correct "solution" for Levers simulation

References

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Vectorpark, Inc. (2016). *Levers*. Retrieved from http://www.vectorpark.com/levers/

Lesson Plan for 2016-11-07

Class	AP Physics C: Mechanics
Title/Topic	Rotational Dynamics
Standards	- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force. - E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations. - E.3.c.2: Determine the angular acceleration with which a rigid object is accelerated about a fixed axis when subjected to a specified external torque or force. - E.3.c.3: Determine the radial and tangential acceleration of a point on a rigid object. - E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation. - E.3.c.5: Analyze problems involving strings and massive pulleys. - E.3.d.1: Write down, justify and apply the relation between linear and angular velocity, or between linear and angular acceleration, for an object of circular cross-section that rolls without slipping along a fixed plane, and determine the velocity and acceleration of an arbitrary point on such an object. - E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion. - E.4.a.1: Calculate the torque of a specified force about an arbitrary origin.
Objectives	Students should be able to: - Determine the effect of multiple torques on the rotation of an object. - Perform calculations on rotational dynamics problems.
Context	1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with information on rotational statics and dynamics that have been incorporated into the model. 2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations on rotational statics and dynamics problems.
Materials	- TIPERs – B6-QRT07, B6-QRT09 - Practice – Rotational Dynamics
Safety	N/A

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (35 minutes) Rigid Body Rotation Model
	    - What are the parts of the rigid body rotation model that we know so far?
	    - How does the rigid body rotation model encompass the laboratory work and practice?
	- (10 minutes) TIPERs – B6-QRT07, B6-QRT09
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?
	- (45 minutes) Practice – Rotational Dynamics
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?

Differentiation	- Help students who are struggling with calculations - Challenge students who have finished to help their peers
Assessment	- Model creation - TIPERs - Calculations on problems
References	Hieggelke, C. J., Kanim, S., Maloney, D. P., & O'Kuma, T. L. (2015). *TIPERs: Sensemaking tasks for introductory physics*. Boston, MA: Pearson. Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-09

Class AP Physics C: Mechanics

Title/Topic Rotational Statics and Dynamics

Standards
- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force.
- E.2.a.2: Calculate the torque on a rigid object due to gravity.
- E.2.b.1: State the conditions for translational and rotational equilibrium of a rigid object.
- E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations.
- E.2.e: Students should be able to state and apply the parallel-axis theorem.
- E.3.c.2: Determine the angular acceleration with which a rigid object is accelerated about a fixed axis when subjected to a specified external torque or force.
- E.3.c.3: Determine the radial and tangential acceleration of a point on a rigid object.
- E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation.
- E.3.c.5: Analyze problems involving strings and massive pulleys.
- E.3.d.1: Write down, justify and apply the relation between linear and angular velocity, or between linear and angular acceleration, for an object of circular cross-section that rolls without slipping along a fixed plane, and determine the velocity and acceleration of an arbitrary point on such an object.
- E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.
- E.4.a.1: Calculate the torque of a specified force about an arbitrary origin.

Objectives Students should be able to:
- Perform calculations on rotational static problems.
- Perform calculations on rotational dynamics problems.

Context
1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with information on rotational statics and dynamics that have been incorporated into the model.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations on rotational statics and dynamics problems.

Materials	- Practice – Rotational Statics and Dynamics
Safety	N/A

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (90 minutes) Practice – Rotational Statics and Dynamics
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?

Differentiation	- Help students who are struggling with calculations - Challenge students who have finished to help their peers
Assessment	- Calculations on problems
References	Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-10

Class	AP Physics C: Mechanics
Title/Topic	Rotational Statics and Dynamics

Standards

- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force.
- E.2.a.2: Calculate the torque on a rigid object due to gravity.
- E.2.b.1: State the conditions for translational and rotational equilibrium of a rigid object.
- E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations.
- E.2.e: Students should be able to state and apply the parallel-axis theorem.
- E.3.c.2: Determine the angular acceleration with which a rigid object is accelerated about a fixed axis when subjected to a specified external torque or force.
- E.3.c.3: Determine the radial and tangential acceleration of a point on a rigid object.
- E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation.
- E.3.c.5: Analyze problems involving strings and massive pulleys.
- E.3.d.1: Write down, justify and apply the relation between linear and angular velocity, or between linear and angular acceleration, for an object of circular cross-section that rolls without slipping along a fixed plane, and determine the velocity and acceleration of an arbitrary point on such an object.
- E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.
- E.4.a.1: Calculate the torque of a specified force about an arbitrary origin.

Objectives

Students should be able to:

- Perform calculations on rotational static problems.
- Perform calculations on rotational dynamics problems.

Context

1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with information on rotational statics and dynamics that have been incorporated into the model.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations on rotational statics and dynamics problems.

Materials	- Practice problem - Quiz – Rotational Statics and Dynamics
Safety	N/A

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (35 minutes) Practice problem
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?
	- (55 minutes) Quiz – Rotational Statics and Dynamics
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?

Differentiation	- Help students who are struggling with calculations - Challenge students who have finished problem to help their peers - Accommodations on quiz as determined by 504/IEP
Assessment	- Quiz
References	Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-11

Class	AP Physics C: Mechanics
Title/Topic	Angular Momentum
Standards	- E.4.a.2: Calculate the angular momentum vector for a moving particle. - E.4.a.3: Calculate the angular momentum vector for a rotating rigid object in simple cases where this vector lies parallel to the angular velocity vector.
Objectives	Students should be able to: - Determine the relationship between moment of inertia and angular velocity. - Create and interpret a graph of the data. - Describe the mathematical relationship between angular momentum, moment of inertia, and angular velocity.
Context	1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because ideas about angular momentum are foundational to rigid body rotation. 2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to determine the relationship between moment of inertia and angular velocity. 3. *Students will utilize laboratory skills to collect data, interpret data, and create conclusions*. The lesson meets this goal because students are collecting data, interpreting data, and creating conclusions about the relationship between the moment of inertia and angular velocity.
Materials	- Rotary motion sensor – 1 set per group - Ring stands – 1 per group - LabQuest 2 (from Vernier) – 1 per group - Rotational motion accessory kit – 1 per group - Whiteboards – 1 or 2 per group - Markers and towels – class set
Safety	Handle equipment appropriately

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (60 minutes) Lab – Angular Momentum
	    - How does the moment of inertia affect the angular velocity?
	    - What does the slope of a graph of the data represent?
	- (15 minutes) Group discussion of results of Lab – Angular Momentum
	    - What is your interpretation of the data and analysis?
	    - What could have gone wrong in this lab?

- (15 minutes) Discussion – Angular Momentum
  - In what ways can we represent angular momentum?
  - How does angular momentum relate to other parts of the rigid body rotation model?

Differentiation
- Help students who are struggling in the lab
- Challenge students who complete lab quickly to provide interpretation of other situations

Assessment
- Collection of data, graphing, and interpretations
- Individual students in the whole-group discussion

References

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-14

Class	AP Physics C: Mechanics
Title/Topic	Angular Momentum
Standards	- E.4.a.3: Calculate the angular momentum vector for a rotating rigid object in simple cases where this vector lies parallel to the angular velocity vector. - E.4.b.2: State the relation between net external torque and angular momentum, and identify situations in which angular momentum is conserved. - E.4.b.3: Analyze problems in which the moment of inertia of an object is changed as it rotates freely about a fixed axis.
Objectives	Students should be able to: - Determine the change in angular momentum when one object is dropped on a spinning object. - Perform calculations from experimental data. - Determine the amount of energy lost when the object is dropped.
Context	1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because ideas about the conservation of angular momentum are foundational to rigid body rotation. 2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to determine the change in angular momentum in the initial and final conditions and the amount of energy lost between the two conditions. 3. *Students will utilize laboratory skills to collect data, interpret data, and create conclusions*. The lesson meets this goal because students are collecting data and performing calculations based on the data.
Materials	- Lab – Conservation of Angular Momentum - Rotary motion sensor – 1 set per group - Ring stands – 1 per group - LabQuest 2 (from Vernier) – 1 per group - Rotational motion accessory kit – 1 per group - Whiteboards – 1 or 2 per group - Markers and towels – class set - Practice – Angular Momentum
Safety	Handle equipment appropriately
Instructional Sequence	- (7 minutes) Greeting and general discussion - (75 minutes) Lab – Conservation of Angular Momentum

- How does the moment of inertia affect the angular velocity?
- What does the slope of a graph of the data represent?
- (15 minutes) Group discussion of results of Lab – Angular Momentum
    - What is your interpretation of the data and analysis?
    - What could have gone wrong in this lab?
- (Homework) Practice – Angular Momentum
    - How do we perform the calculations correctly?
    - Is there another way to solve this problem?

Differentiation

- Help students who are struggling in the lab
- Challenge students who complete lab quickly to provide interpretation of other situations

Assessment

- Collection of data, graphing, and interpretations
- Individual students in the whole-group discussion

References

Dukerich, L. (2011). *Advanced physics with Vernier: Mechanics*. Beaverton, OR: Vernier Software and Technology. Retrieved from https://www.vernier.com/experiments/phys-am/14/conservation_of_angular_momentum/

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-15

Class	AP Physics C: Mechanics
Title/Topic	Angular Momentum
Standards	- E.4.a.3: Calculate the angular momentum vector for a rotating rigid object in simple cases where this vector lies parallel to the angular velocity vector. - E.4.b.1: Recognize the conditions under which the law of conservation is applicable and relate this law to one- and two-particle systems such as satellite orbits. - E.4.b.2: State the relation between net external torque and angular momentum, and identify situations in which angular momentum is conserved. - E.4.b.3: Analyze problems in which the moment of inertia of an object is changed as it rotates freely about a fixed axis. - E.4.b.4: Analyze a collision between a moving particle and a rigid object that can rotate about a fixed axis or about its center of mass.
Objectives	Students should be able to: - Perform calculations for problems with angular momentum of an individual particle or object. - Perform calculations for problems with conservation of angular momentum.
Context	1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with information on angular momentum that have been incorporated into the model. 2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to determine aspects of conservation of angular momentum.
Materials	- Practice – Angular Momentum
Safety	N/A

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (90 minutes) Practice – Angular Momentum
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?

Differentiation	- Help students who are struggling with calculations - Challenge students who complete calculations to help peers

Assessment
- Collection of data, graphing, and interpretations
- Calculations on problems

References

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

Lesson Plan for 2016-11-16

Class
AP Physics C: Mechanics

Title/Topic
Angular Momentum

Standards

- E.4.a.3: Calculate the angular momentum vector for a rotating rigid object in simple cases where this vector lies parallel to the angular velocity vector.
- E.4.b.2: State the relation between net external torque and angular momentum, and identify situations in which angular momentum is conserved.
- E.4.b.3: Analyze problems in which the moment of inertia of an object is changed as it rotates freely about a fixed axis.
- E.4.b.3: Analyze a collision between a moving particle and a rigid object that can rotate about a fixed axis or about its center of mass.
- E.4.b.4: Analyze a collision between a moving particle and a rigid object that can rotate about a fixed axis or about its center of mass.

Objectives

Students should be able to:

- Perform calculations for problems with angular momentum of an individual particle or object.
- Perform calculations for problems with conservation of angular momentum.

Context

1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with information on angular momentum that have been incorporated into the model.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to determine aspects of conservation of angular momentum.

Materials

- Practice – Conservation and Total Angular Momentum
- Quiz – Angular Momentum

Safety
N/A

Instructional Sequence

- (7 minutes) Greeting and general discussion
- (45 minutes) Practice – Conservation and Total Angular Momentum
  - How do we perform the calculations correctly?
  - Is there another way to solve this problem?
- (45 minutes) Quiz – Angular Momentum
  - How do we perform the calculations correctly?
  - Is there another way to solve this problem?

Differentiation	-	Help students who are struggling with calculations
	-	Challenge students who complete calculations to help peers
	-	Accommodations on quiz as determined by 504/IEP
Assessment	-	Practice problems
	-	Quiz

References

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.

OpenStax (2016). *University physics volume 1*. Retrieved from http://cnx.org/content/col12031/latest/

Lesson Plan for 2016-11-17

Class	AP Physics C: Mechanics
Title/Topic	Review of Rigid Body Rotation

Standards

- D.1.a.1: Identify by inspection the center of mass of a symmetrical object.
- D.1.a.2: Locate the center of mass of a system consisting of two such objects.
- D.1.b: Students should be able to understand and apply the relation between center-of-mass velocity and linear momentum, and between center-of-mass acceleration and net external force for a system of particles.
- D.1.c: Students should be able to define center of gravity and to use this concept to express the gravitational potential energy of a rigid object in terms of the position of its center of mass.
- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force.
- E.2.a.2: Calculate the torque on a rigid object due to gravity.
- E.2.b.1: State the conditions for translational and rotational equilibrium of a rigid object.
- E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations.
- E.2.c.1: Determine by inspection which of a set of symmetrical objects of equal mass has the greatest rotational inertia.
- E.2.c.2: Determine by what factor an object’s rotational inertia changes if all its dimensions are increased by the same factor.
- E.2.d.1: A collection of point masses lying in a plane about an axis perpendicular to the plane.
- E.2.d.2: A thin rod of uniform density, about an arbitrary axis perpendicular to the rod.
- E.2.d.3: A thin cylindrical shell about its axis, or an object that may be viewed as being made up of coaxial shells.
- E.2.e: Students should be able to state and apply the parallel-axis theorem.
- E.3.b: Students should be able to use the right-hand rule to associate an angular velocity vector with a rotating object.
- E.3.c.1: Describe in detail the analogy between fixed-axis rotation and straight-line translation.
- E.3.c.2: Determine the angular acceleration with which a rigid object is accelerated about a fixed axis when subjected to a specified external torque or force.
- E.3.c.3: Determine the radial and tangential acceleration of a point on a rigid object.

- E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation.
- E.3.c.5: Analyze problems involving strings and massive pulleys.
- E.3.d.1: Write down, justify and apply the relation between linear and angular velocity, or between linear and angular acceleration, for an object of circular cross-section that rolls without slipping along a fixed plane, and determine the velocity and acceleration of an arbitrary point on such an object.
- E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.
- E.4.a.1: Calculate the torque of a specified force about an arbitrary origin.
- E.4.a.2: Calculate the angular momentum vector for a moving particle.
- E.4.a.3: Calculate the angular momentum vector for a rotating rigid object in simple cases where this vector lies parallel to the angular velocity vector.
- E.4.b.1: Recognize the conditions under which the law of conservation is applicable and relate this law to one- and two-particle systems such as satellite orbits.
- E.4.b.2: State the relation between net external torque and angular momentum, and identify situations in which angular momentum is conserved.
- E.4.b.3: Analyze problems in which the moment of inertia of an object is changed as it rotates freely about a fixed axis.
- E.4.b.4: Analyze a collision between a moving particle and a rigid object that can rotate about a fixed axis or about its center of mass.

Objectives

Students should be able to:
- Perform calculations for problems with rotational statics.
- Perform calculations for problems with rotational dynamics.
- Perform calculations for problems with angular momentum of an individual particle or object.
- Perform calculations for problems with conservation of angular momentum.

Context

1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with all information that has been incorporated into the model.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to all aspects of rigid body rotation.

Materials

- Review – Rotation II (Rigid Body Rotation)

Safety

N/A

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (15 minutes) Rigid Body Rotation Model Update
	    - What are the parts of the complete rigid body rotation model?
	    - How does the rigid body rotation model encompass the laboratory work and practice?
	- (75 minutes) Review – Rotation II (Rigid Body Rotation)
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?

Differentiation	- Help students who are struggling with calculations - Challenge students who complete calculations to help peers
Assessment	- Review
References	Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson. OpenStax (2016). *University physics volume 1*. Retrieved from http://cnx.org/content/col12031/latest/

Lesson Plan for 2016-11-18

Class	AP Physics C: Mechanics
Title/Topic	Test for Rigid Body Rotation

Standards

- D.1.a.1: Identify by inspection the center of mass of a symmetrical object.
- D.1.a.2: Locate the center of mass of a system consisting of two such objects.
- D.1.b: Students should be able to understand and apply the relation between center-of-mass velocity and linear momentum, and between center-of-mass acceleration and net external force for a system of particles.
- D.1.c: Students should be able to define center of gravity and to use this concept to express the gravitational potential energy of a rigid object in terms of the position of its center of mass.
- E.2.a.1: Calculate the magnitude and direction of the torque associated with a given force.
- E.2.a.2: Calculate the torque on a rigid object due to gravity.
- E.2.b.1: State the conditions for translational and rotational equilibrium of a rigid object.
- E.2.b.2: Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations.
- E.2.c.1: Determine by inspection which of a set of symmetrical objects of equal mass has the greatest rotational inertia.
- E.2.c.2: Determine by what factor an object's rotational inertia changes if all its dimensions are increased by the same factor.
- E.2.d.1: A collection of point masses lying in a plane about an axis perpendicular to the plane.
- E.2.d.2: A thin rod of uniform density, about an arbitrary axis perpendicular to the rod.
- E.2.d.3: A thin cylindrical shell about its axis, or an object that may be viewed as being made up of coaxial shells.
- E.2.e: Students should be able to state and apply the parallel-axis theorem.
- E.3.b: Students should be able to use the right-hand rule to associate an angular velocity vector with a rotating object.
- E.3.c.1: Describe in detail the analogy between fixed-axis rotation and straight-line translation.
- E.3.c.2: Determine the angular acceleration with which a rigid object is accelerated about a fixed axis when subjected to a specified external torque or force.
- E.3.c.3: Determine the radial and tangential acceleration of a point on a rigid object.

- E.3.c.4: Apply conservation of energy to problems of fixed-axis rotation.
- E.3.c.5: Analyze problems involving strings and massive pulleys.
- E.3.d.1: Write down, justify and apply the relation between linear and angular velocity, or between linear and angular acceleration, for an object of circular cross-section that rolls without slipping along a fixed plane, and determine the velocity and acceleration of an arbitrary point on such an object.
- E.3.d.3: Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.
- E.4.a.1: Calculate the torque of a specified force about an arbitrary origin.
- E.4.a.2: Calculate the angular momentum vector for a moving particle.
- E.4.a.3: Calculate the angular momentum vector for a rotating rigid object in simple cases where this vector lies parallel to the angular velocity vector.
- E.4.b.1: Recognize the conditions under which the law of conservation is applicable and relate this law to one- and two-particle systems such as satellite orbits.
- E.4.b.2: State the relation between net external torque and angular momentum, and identify situations in which angular momentum is conserved.
- E.4.b.3: Analyze problems in which the moment of inertia of an object is changed as it rotates freely about a fixed axis.
- E.4.b.4: Analyze a collision between a moving particle and a rigid object that can rotate about a fixed axis or about its center of mass.

Objectives

Students should be able to:
- Perform calculations for problems with rotational statics.
- Perform calculations for problems with rotational dynamics.
- Perform calculations for problems with angular momentum of an individual particle or object.
- Perform calculations for problems with conservation of angular momentum.

Context

1. *Students will develop a coherent model for rigid body rotation*. The lesson meets this goal because students are practicing with all information that has been incorporated into the model.

2. *Students will perform calculations in rigid body rotation*. The lesson meets this goal because students perform calculations to all aspects of rigid body rotation.

Materials

- Test – Rotation II

Safety

N/A

Instructional Sequence	- (7 minutes) Greeting and general discussion
	- (90 minutes) Test – Rotation II (Rigid Body Rotation)
	    - How do we perform the calculations correctly?
	    - Is there another way to solve this problem?

Differentiation	- Accommodations on test as determined by 504/IEP
Assessment	- Test
References	Chabay, R. W. & Sherwood, B. A. (2015). *Matter & interactions* (4th ed.). Hoboken, NJ: John Wiley and Sons. Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson. OpenStax (2016). *University physics volume 1*. Retrieved from http://cnx.org/content/col12031/latest/

Models in AP Physics C:

<u>Definitions:</u>

- Model: Representation of structure in a given system
- System: Set of related objects, which may be real or imaginary, physical or mental, simple or composite
- Structure: Set of relations among its objects

<u>Parts of a Model:</u>

- Descriptions:
    - Object Description:
        - Type
        - Composition
        - Object variables – Represent intrinsic properties of the object have fixed values
    - Process Description:
        - Reference system
        - State variables – Represent intrinsic properties with values that may vary with time; a descriptor regarded as state variables in one model may be an object variable in another model
        - Often useful to use graphical methods
    - Interaction Description:
        - Type and agent
        - Interaction variables – Represents the interaction of some external object (called an agent) with the object being modeled
        - Often useful to use diagrams
- Formulations:
    - Dynamical Laws – Mathematical equation(s) that determine(s) the time evolution of state variables
    - Interaction Laws – Mathematical equation(s) that express(es) interaction variables as functions of state variables
- Ramifications:
    - Linguistic – Written and verbal communication about the system and structure
    - Computational – Use of a computer program to encode the system and structure

Rigid Body Rotation Model

1. Descriptions:
    a. Object Description
        i. Object variables
            1. Angular momentum
            2. Rotational kinetic energy
            3. Torque
    b. Process Description
        i. State variables
            1. Angle
            2. Angular velocity
            3. Angular acceleration
            4. Center of mass
            5. Moment of inertia
            6. Radius
    c. Interaction Description
        i. Diagrams
            1. Force diagram
            2. Free-body diagram
            3. Energy chart (LOL diagram)
2. Formulations:
    a. Interaction Laws
        i. $\vec{\tau} = \vec{r} \times \vec{F}$
        ii. $\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$
        iii. $I = \int r^2 dm = \sum m r^2$
        iv. $x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$
        v. $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$
        vi. $K = \frac{1}{2} I \omega^2$
3. Ramifications:
    a. Every object has a center of mass, but this point may not be in the geometric middle of the object.
    b. Moment of inertia of an object is related to the shape and orientation of the object.
    c. Total kinetic energy of an object is the sum of translational kinetic energy and rotational kinetic energy.
    d. The torque an object experiences is related to where and how forces are applied.
    e. An object achieves equilibrium when the net force in all directions and net torque in all directions equal zero.
    f. Angular momentum of an object is related to the moment of inertia and angular velocity of the object.
    g. Angular momentum may be conserved for a given situation.

Part III: Assessing Student Learning

The assessment plan for this curriculum unit contains formative assessments embedded in each lesson and a written summative assessment at the end of the unit. Formative assessments include laboratory work, written practice, and written quizzes, which allow students and the teacher to gauge the transformation of the rigid body rotation mental model to the accepted scientific rigid body rotation conceptual model. The summative assessment is a written assessment with conceptual and calculation multiple-choice questions and calculation free-response questions. The formative assessments align with Goal 1 because each is working to develop or refine the rigid body rotation mental model of each student, and the assessments align with Goal 2 because students practice calculations during laboratory work and instruction. Goal 3 is to help students develop laboratory skills, and the teacher provided feedback to students during laboratory work to question the methods used by students, determine mistakes in calculations, discuss results from data collection, and create meaningful interpretations of the analysis. The summative assessment best aligns with Goals 1 and 2 because the assessment focused more on aspects of the model and calculations.

Date	**Type of Assessment**	**Name of Laboratory or Practice**	**Instructional Goal Alignment**
2016-10-21	Formative	Lab – Center of Mass	1, 2, 3
2016-10-24	Formative	Lab – Rotational Kinetic Energy	1, 2, 3
2016-10-25	Formative	TIPERs	1, 2
2016-10-25	Formative	Practice – Center of Mass, Moment of Inertia, and Rotational Energy	1, 2
2016-10-26	Formative	Practice – Center of Mass, Moment of Inertia, and Rotational Energy	1, 2
2016-10-27	Formative	TIPERs	1, 2
2016-10-27	Formative	Practice – Problem on Presentation	1, 2
2016-10-27	Summative and Formative	Quiz – Center of Mass, Moment of Inertia, and Rotational Energy	1, 2

Date	Type of Assessment	Name of Laboratory or Practice	Instructional Goal Alignment
2016-10-28	Formative	Lab – Rotational Statics	1, 2, 3
2016-10-31	Formative	Practice – Cross Products and Rotational Statics	1, 2
2016-10-31	Formative	TIPERs	1, 2
2016-11-01	Formative	Lab – Rotational Dynamics	1, 2, 3
2016-11-02	Formative	Practice – Cross Products and Rotational Statics	1, 2
2016-11-03	Formative	TIPERs	1, 2
2016-11-03	Formative	AP Physics C: Mechanics Free-Response Problems	1, 2
2016-11-04	Formative	AP Physics C: Mechanics Free-Response Problems	1, 2
2016-11-04	Formative	Lab – Levers	1, 3
2016-11-07	Formative	TIPERs	1, 2
2016-11-07	Formative	Practice – Rotational Dynamics	1, 2
2016-11-09	Formative	Practice – Rotational Statics and Dynamics	1, 2
2016-11-10	Formative	Practice – Problem on Presentation	1, 2
2016-11-10	Summative and Formative	Quiz – Rotational Statics and Dynamics	1, 2
2016-11-11	Formative	Lab – Angular Momentum	1, 2, 3
2016-11-14	Formative	Lab – Conservation of Angular Momentum	1, 2, 3
2016-11-14	Formative	Practice – Angular Momentum	1, 2
2016-11-15	Formative	Practice – Angular Momentum	1, 2
2016-11-16	Formative	Practice – Conservation and Total Angular Momentum	1, 2
2016-11-16	Summative and Formative	Quiz – Angular Momentum	1, 2
2016-11-17	Formative	Review – Rotation II (Rigid Body Rotation)	1, 2
2016-11-18	Summative	Test – Rotation II (Rigid Body Rotation)	1, 2

Part IV: Resources
Instructional materials, notes, and assessments were included after each lesson plan.

Blog of Phyz. (2016). *Dizzying dance demonstrates dynamics*. Retrieved from http://phyzblog.blogspot.com/2016/06/dizzying-dance-demonstrates-dynamics.html?m=1
Video of a dancer using a cyr wheel, balancing herself in the wheel as she moves around the floor. This technique requires incredible balance and understanding of rotation, and the researcher and students were impressed with the dancer's fluidity and power.

Boas, M. L. (2006). *Mathematical methods in the physical sciences* (3rd ed.). Hoboken, NJ: John Wiley and Sons.
Mathematics textbook for students studying physics; used to present information on cross products.

Chabay, R. W. & Sherwood, B. A. (2015). *Matter & interactions* (4th ed.). Hoboken, NJ: John Wiley and Sons.
Physics textbook written in a different manner than many physics textbooks because it contains many modern physics concepts. In addition, the order of material in the mechanics section is significantly different than most textbooks.

College Board (2015). *Rotational motion curriculum module*. Retrieved from https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-physics-1-curriculum-module-2015-ada.pdf
Resource on rotational motion produced by the College Board for AP Physics 1, and this information was useful to determine laboratory work for students.

Dukerich, L. (2011). *Advanced physics with Vernier: Mechanics*. Beaverton, OR: Vernier Software and Technology.
Laboratory materials from Vernier, which helped determine laboratory work for students.

Hieggelke, C. J., Kanim, S., Maloney, D. P., & O'Kuma, T. L. (2015). *TIPERs: Sensemaking tasks for introductory physics*. Boston, MA: Pearson.
Tasks Inspired by Physics Education Research (TIPERs) are short conceptual questions that require students to think deeply about a concept to answer the question. TIPERs have several different types, including ranking tasks, descriptions, and correcting given reasoning about a concept.

Knight, R. D. (2017a). *Instructor's guide: Teaching with* physics for scientists and engineers (4th ed.). Retrieved from https://www.pearsonhighered.com/product/Knight-Instructor-s-Guide-Download-Only-for-Physics-for-Scientists-and-Engineers-A-Strategic-Approach-with-Modern-Physics-4th-Edition/9780134092485.html
Commentary from Knight about strategies to teach rigid body rotation; more useful for the collegiate level than high school level.

Knight, R. D. (2017b). *Physics for scientists and engineers: A strategic approach* (4th ed.). Boston, MA: Pearson.
Physics textbook written in a more traditional manner, with emphasis on models.

Lane, A. (2011). *Circonvolution improbable*. Available from https://www.youtube.com/watch?v=MBBypvakUdI&list=PLGppBVVrPXke2PZgRsBbaMHpEc3VEIos2&index=3
Video of a dancer using a cyr wheel, demonstrating strength and an impressive understanding of the wheel and his body.

OpenStax (2016). *University physics volume 1*. Retrieved from http://cnx.org/content/col12031/latest/
Physics textbook written in a more traditional manner, and it is a free textbook through Rice University.

Vectorpark, Inc. (2016). *Levers*. Retrieved from http://www.vectorpark.com/levers/
This is an online "game" that forces students to balance many objects above water for a given amount of time. Students are successful when they balance all objects above the water.