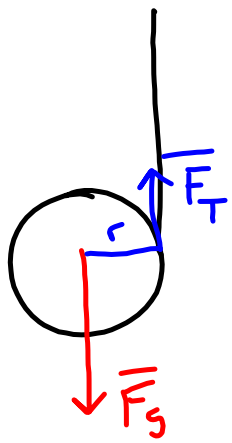
Practice – Angular Momentum

1)



$\downarrow +$

$\sum F_y = ma$

$F_g - F_T = ma_{cm}$

$a_{cm} = \frac{ma_g - F_T}{m}$

$= \frac{ma_g - \frac{I\alpha}{r}}{m}$

$a_{cm} = r\alpha \quad \alpha = \frac{a_{cm}}{r}$

$F_T = ma_g - ma_{cm}$

$\tau = r F_T \cancel{\sin\theta^{1}}$

$\tau = I\alpha$

$I\alpha = r F_T$

$F_T = \frac{I\alpha}{r}$

$= \frac{I a_{cm}}{r^2}$

$\frac{I a_{cm}}{r^2} = ma_g - ma_{cm}$

$I = \frac{1}{2} m r^2$

$\frac{\frac{1}{2} \cancel{m} \cancel{r^2} a_{cm}}{\cancel{r^2}} = \cancel{ma_g} - \cancel{ma_c}$

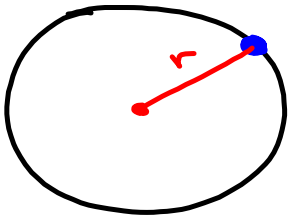
$\frac{1}{2} a_{cm} = a_g - a_c$

$\frac{1}{2} a_{cm} + a_{cm} = a_g$

$\frac{3}{2} a_{cm} = a_g$

$a_{cm} = \frac{2}{3} a_g$

2)



$$I_{total} = I_{disk} + I_{mass}$$

$$= \frac{1}{2} m_d r^2 + m_m r_m^2$$

$$L_i = L_f$$

$$(I_d + I_m)\omega_i = I_d \omega_f$$

$$\omega_f = \frac{(I_d + I_m)\omega_i}{I_d}$$

$$= \frac{(0.36\ kg\cdot m^2 + 0.018\ kg\cdot m^2)(2.0\ rev/s)}{0.36\ kg\cdot m^2}$$

$$= 2.1\ rev/s$$

3) $I = \frac{1}{12}(240\ kg)(6\ m)^2$

$= 720\ kg \cdot m^2$

$1200\ rpm = 125.7\ rad/s$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{125.7\ rad/s - 0\ rad/s}{30\ s} = 4.19\ rad/s^2$$

a. $\omega(10s) = \alpha t$

$= (4.19\ rad/s^2)(10\ s)$

$= 41.9\ rad/s$

$L(10\ s) = I\omega$

$= (720\ kg \cdot m^2)(41.9\ rad/s)$

$= 30168\ kg \cdot m^2/s$

$\omega(20\ s) = 83.8\ rad/s$

$L(20\ s) = 60336\ kg \cdot m^2/s$

b. $$\tau = \frac{\Delta L}{\Delta t} = \frac{60336\ kg \cdot m^2/s - 30168\ kg \cdot m^2/s}{10\ s}$$

$= 3017\ N \cdot m$

4)

$$\bar{L}_i = \bar{L}_f$$

$$I_m \omega_i = (I_m + I_b)\omega_f$$

$$\omega_f = \frac{I_m \omega_i}{I_m + I_b}$$

$$= \frac{\frac{1}{2} m_m r^2 \omega_i}{\frac{1}{2} m_m r^2 + m_b r^2}$$

$$= \frac{\frac{1}{2}(120\,kg)(1.6\,m)^2 (0.5\,rev/s)}{\frac{1}{2}(120\,kg)(1.6\,m)^2 + \frac{1}{2}(22\,kg)(1.6\,m)^2}$$

$$= 0.366\ rev/s$$

5) a. $\vec{L}_i = \vec{L}_f$

$$r m_p v_p = (I_d + I_p)\omega_f$$

$$\omega_f = \frac{r m_p v_p}{I_d + I_p}$$

$$= \frac{r m_p v_p}{\frac{1}{2} m_d r^2 + m_p r^2}$$

$$= \frac{(0.1\,m)(0.02\,kg)(10\,m/s)}{\frac{1}{2}(0.5\,kg)(0.1\,m)^2 + (0.02\,kg)(0.1\,m)^2}$$

$$= 7.4\,rad/s$$

b. $\Delta K = K_f - K_i$

$$= \frac{1}{2} I \omega^2 + \frac{1}{2} m v_{pi}^2$$

$$= 0.07\,J - 1.0\,J$$

$$= -0.93\,J$$

6) a. $\vec{L}_i = \vec{L}_f$

$$r m_b v_b = (I_b + I_a)\omega_f$$

$$\omega_f = \frac{r m_b v_b}{I_b + I_a}$$

$$= \frac{(0.5\,m)(0.145\,kg)(40\,m/s)}{(0.145\,kg)(0.5\,m)^2 + \frac{1}{3}(4\,kg)(0.5\,m)^2}$$

$$= 7.8\ rad/s$$

b. $\tau = I\alpha$ $\qquad \alpha = \frac{\omega_f - \omega_i}{t}$ ($\omega_f \to 0$)

$$= (I_b + I_a)\left(\frac{-\omega_i}{t}\right)$$

$$= \left[(0.145\,kg)(0.5\,m)^2 + \tfrac{1}{3}(4\,kg)(0.5\,m)^2\right]\left(\frac{-7.8\,rad/s}{0.3\,s}\right)$$

$$= 9.62\ N\cdot m$$