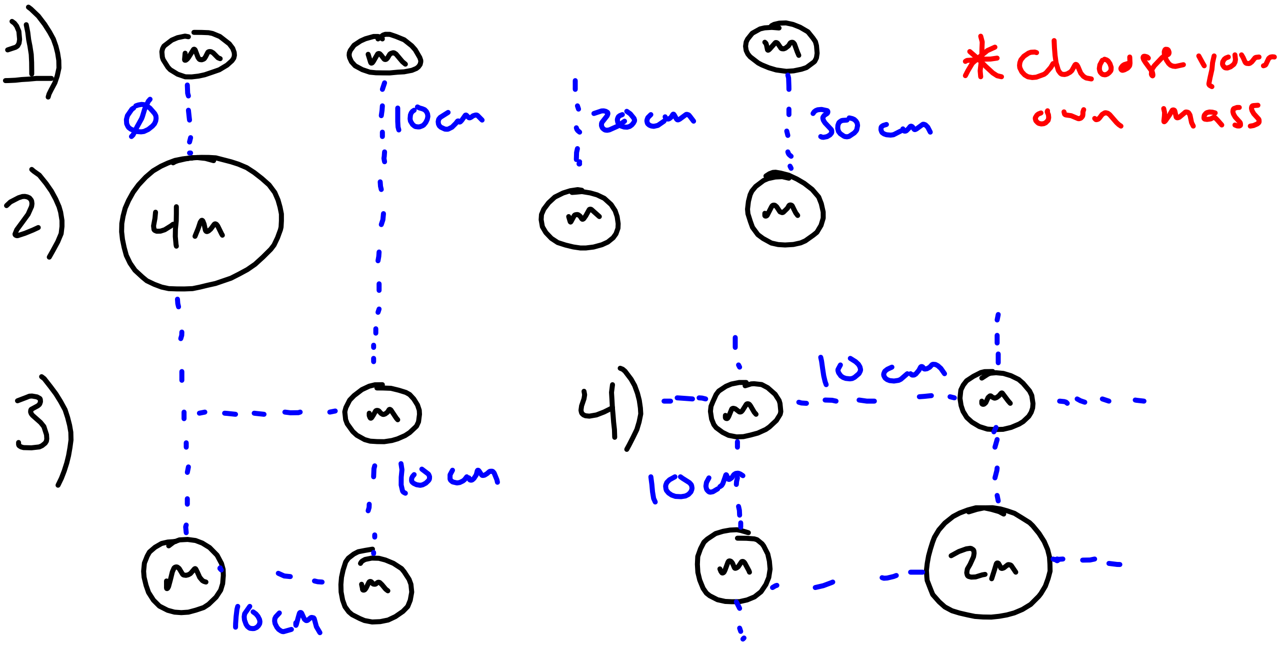
# CENTER OF MASS



# Center of Mass

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$m_i$ ← mass

$x_i$ ← position

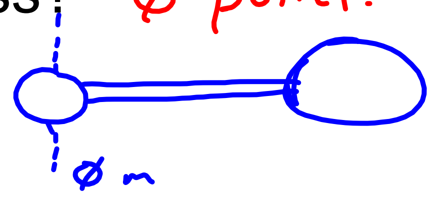
$x_{cm}$ ↑ center of mass in one direction

$\sum m_i$ ↑ sum of masses

1. A barbell consists of a 750-g ball and a 2.5 kg ball connected by a massless 50-cm-long rod.

a) Where is the center of mass?

* You choose ø point.




$$x_{cm} = \frac{\sum_i x_i m_i}{\sum_i m_i}$$

$$= \frac{(\emptyset\ m)(0.75\ kg) + (.50\ m)(2.5\ kg)}{0.75\ kg + 2.5\ kg}$$

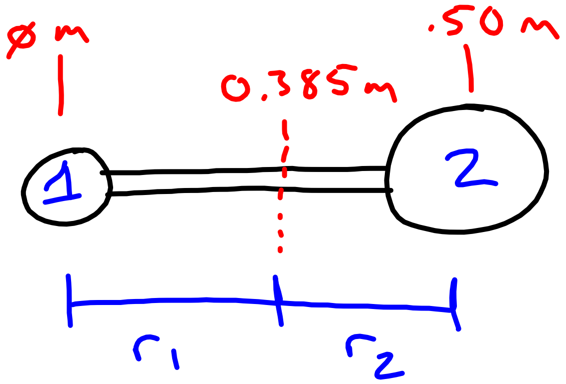
$$= 0.385\ m$$

b) What is the speed of each ball if they rotate about the center of mass at 50 rpm?

$$v = r\omega$$

↳ change to rad/s

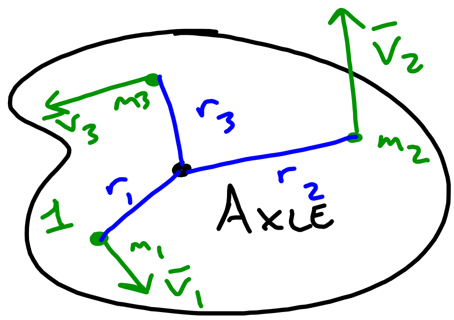
$$\left(50 \frac{rev}{min}\right)\left(\frac{1\ min}{60\ s}\right)\left(\frac{2\pi\ rad}{1\ rev}\right) = 5.236\ rad/s$$




$$v_1 = r_1\omega = (0.385\ m)(5.236\ rad/s) = 2.014\ m/s$$

$$v_2 = r_2\omega = (0.5\ m - 0.385\ m)(5.236\ rad/s) = 0.604\ m/s$$

# Rotational Energy and Moment of Inertia




$$K_{total} = K_{translation} + K_{rotation} + K_{vibration}$$

↑ we ignore

$$K_{rotational} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + \ldots$$

$v = r\omega$

$$= \frac{1}{2}m_1(r_1\omega)^2 + \frac{1}{2}m_2(r_2\omega)^2 + \frac{1}{2}m_3(r_3\omega)^2 + \ldots$$

$$= \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

Moment of Inertia → $I$

$$K_{rot} = \frac{1}{2}I\omega^2$$

$$I \equiv \sum_i m_i r_i^2$$

Ch. 10 in OpenStax "College Physics"
p. 510 → list of common moments of inertia